Charm Meson Dalitz Plot Analyses @ BaBar

Kalanand Mishra

University of Cincinnati

BaBar Collaboration

BaBar results on DP analyses of D and D_s decays:

• $D^0 \rightarrow K^- K^+ \pi^0$	**Phys. Rev. D 76, 011102 (2007)**
• $D^0 \rightarrow \pi^- \pi^+ \pi^0$	**hep-ex / 0703037, submitted to PRL**
• $D^0 \rightarrow K_s^0 \pi^+ \pi^-$	**hep-ex / 0507101 & 0607104**
• $D_s^+ \rightarrow K^+ K^- \pi^+$	**Preliminary**

Preliminary

Hadron 07, INFN Frascati, October 8-13, 2007

3-Particle Phase Space

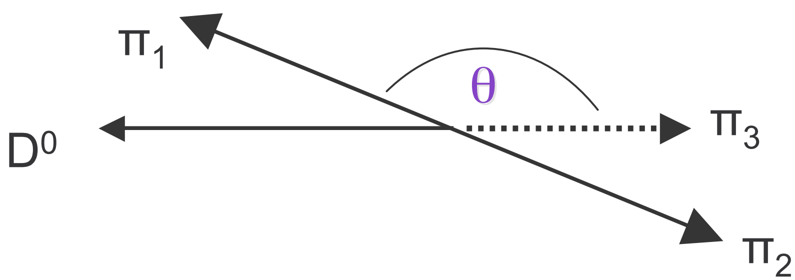
- **2 Observables**

From four vectors	12
Conservation laws	-4
Final state particle masses	-3
Free rotation in decay plane	-3
Σ	2

- **Usual choice**

  Invariant mass squared m^2_{12}

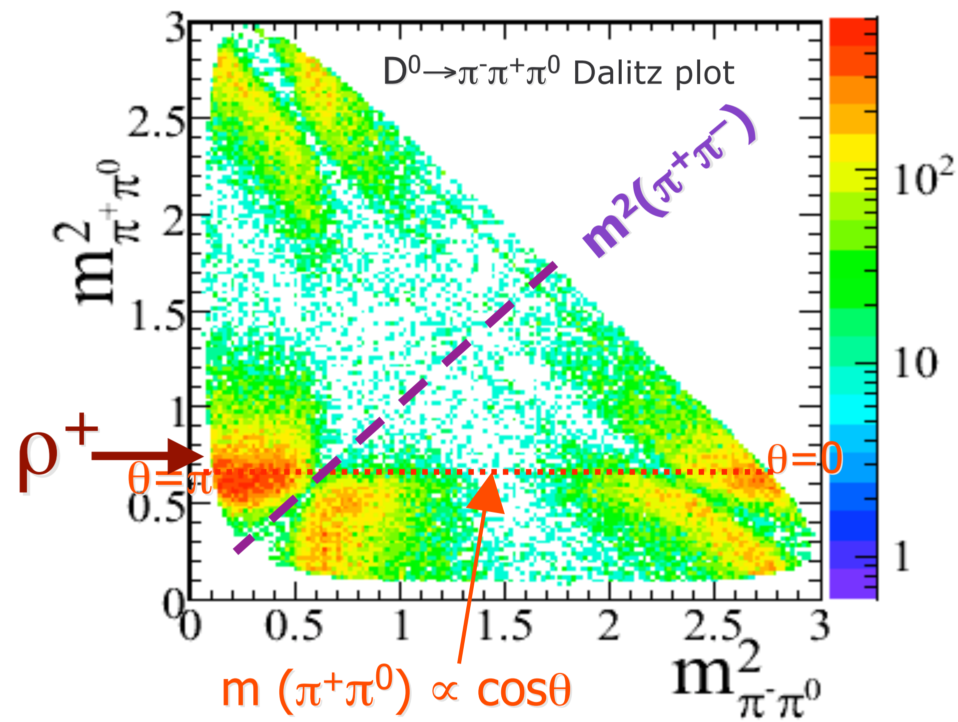
  Invariant mass squared m^2_{13}



$\{\pi_1, \pi_2, \pi_3\} = \{\pi^+, \pi^0, \pi^-\}$

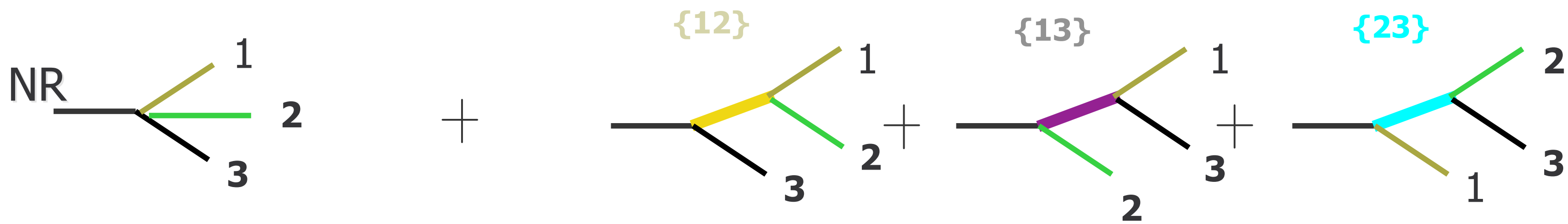
Dalitz plot provides information about two-body amplitude structure.

$$m^2_{\pi^+\pi^0} + m^2_{\pi^-\pi^0} + m^2_{\pi^+\pi^-} = m^2_{\pi^+} + m^2_{\pi^-} + m^2_{\pi^0} + m^2_{D^0}$$



Isobar Model Formalism

three-body decay D→ABC decaying through an [r→AB] resonance



D decay three-body amplitude $$\mathcal{A}_D(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta r} \mathcal{A}_r(s_{12}, s_{13})$$

→ NR term (direct 3 body decay)

a_0, δ_0, a_r, δ_r : Free parameters of fit

$$\mathcal{A}_r(s_{12}, s_{13}) = F_D^J F_r^J \times M_r^J \times BW_r^J$$

Relativistic Breit-Wigner

$$BW_r^J(s) = \begin{cases} \dfrac{1}{M_r^2 - s - iM_r\Gamma_r(\sqrt{s})} \\ \dfrac{1}{M_r^2 - s - iM_r(\rho_1 g_1^2 + \rho_2 g_2^2)} \end{cases}$$

$f_0(980)$
$a_0(980)$

→ Angular distribution

→ *D* and *r* form factors

Introducing Angular Distributions

BABAR™

Schrödinger's Equation

$$-\frac{\hbar}{2\mu}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r})$$

$$\begin{cases} V(\vec{r}) = 0 \\ \vec{k} = \frac{\vec{p}}{\hbar} = \mu\frac{\vec{v}}{\hbar} \quad \mu = \frac{m_1 m_2}{m_1 + m_2} \end{cases}$$

$$|i\rangle = \Psi_i = \sum_{l=0}^{\infty} U_l(r) P_l(\cos\vartheta)$$

$$\Psi_S = \Psi_f - \Psi_i = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\frac{\eta_l e^{2\imath\delta_l}-1}{2\imath} P_l(\cos\vartheta)\frac{e^{\imath kr}}{r}$$

Angular Amplitude: $P_l(\cos\vartheta)$

Dynamic Amplitude (BW, Flatte, S-wave): $(2l+1)\frac{\eta_l e^{2\imath\delta_l}-1}{2\imath}$

In case only l = 0 (S-wave) and 1 (P-wave) amplitudes are present :

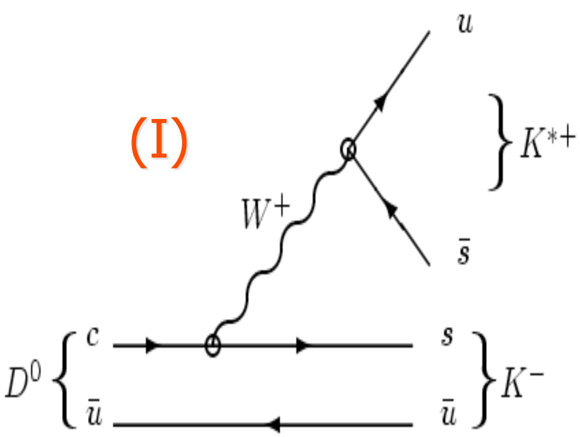
$$\begin{cases} \sqrt{4\pi}\left\langle Y_0^0 \right\rangle = S^2 + P^2 \\ \sqrt{4\pi}\left\langle Y_1^0 \right\rangle = 2|S||P|\cos\phi_{SP} \\ \sqrt{4\pi}\left\langle Y_2^0 \right\rangle = \frac{2}{\sqrt{5}}P^2 \end{cases}$$

For S- and P- waves only, in the absence of cross-feeds from other channels, the amplitudes and the relative phase are given by these relations.

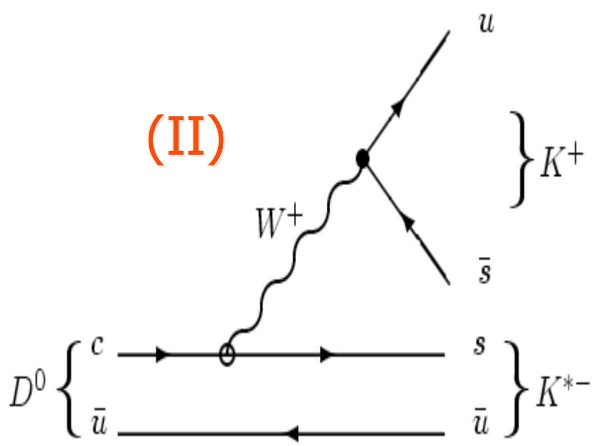
$D^0 \to K^- K^+ \pi^0$ Dalitz Plot Analysis

Interference among three types of singly Cabibbo-suppressed amplitudes

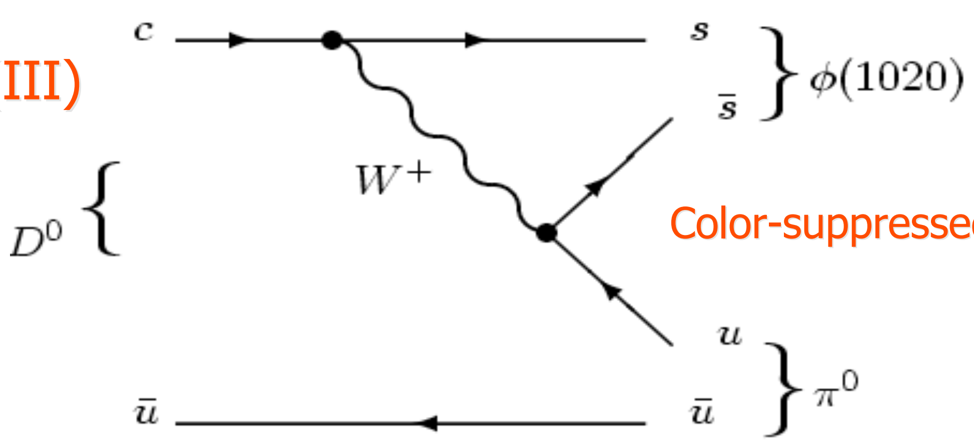
(I)



(II)



(III)

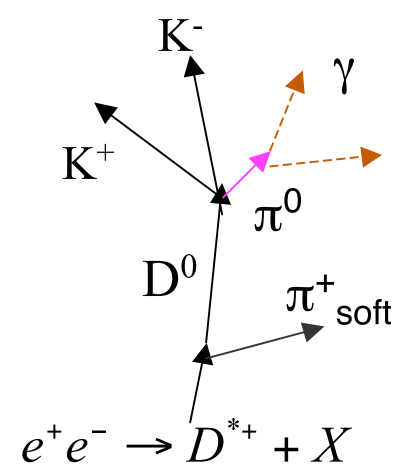


Color-suppressed

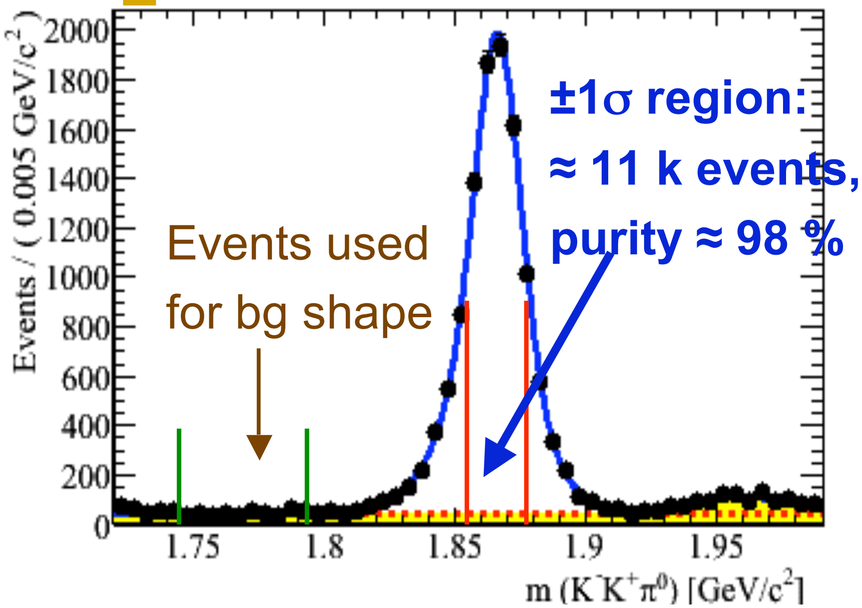
Motivation

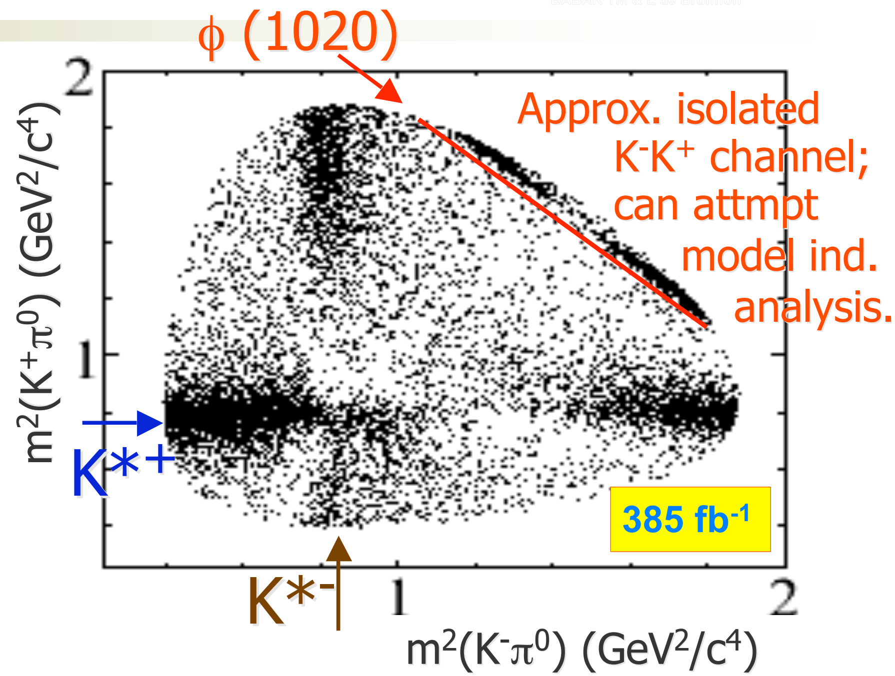
- Nature of $K\pi$ S-wave below 1.4 GeV.
- Is there a charged κ state?
- Nature of f_0/a_0 (980): $K\bar{K}$ S-wave.

D^0 decay reconstruction



Dalitz Plot for $D^0 \to K^- K^+ \pi^0$





Event Selection

$P_{CM}(D^0) > 2.77$ GeV/c
$|m_{D*} - m_{D^0} - 145.5|$
< 0.6 MeV/c^2

Phys. Rev. D74, 091102 (2006)

Define amplitude for the $D^0 \to K^- K^+ \pi^0$ decay as:

$$\mathcal{A}[D^0 \to K^- K^+ \pi^0] \equiv f(m^2_{K^+\pi^0}, m^2_{K^-\pi^0})$$
$$\bar{\mathcal{A}}[\bar{D^0} \to K^+ K^- \pi^0] \equiv f(m^2_{K^-\pi^0}, m^2_{K^+\pi^0})$$

Dalitz plot intensity $\propto |f|^2$

Kπ and K^+K^- *S*-wave Amplitudes

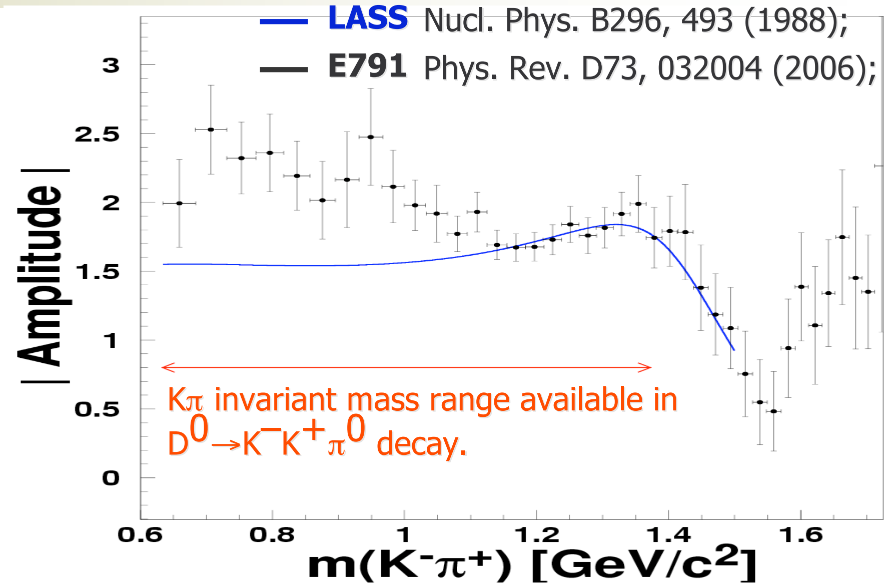
For Kπ S-wave

- The LASS amplitude gives the best fit.
- E-791 fit worse at low mass.

- κ model yields

mass 870 ± 30 MeV/c^2
width 150 ± 20 MeV/c^2

significantly different from the values reported for κ^0.
- κ with E-791 parameters does not give a satisfactory fit.



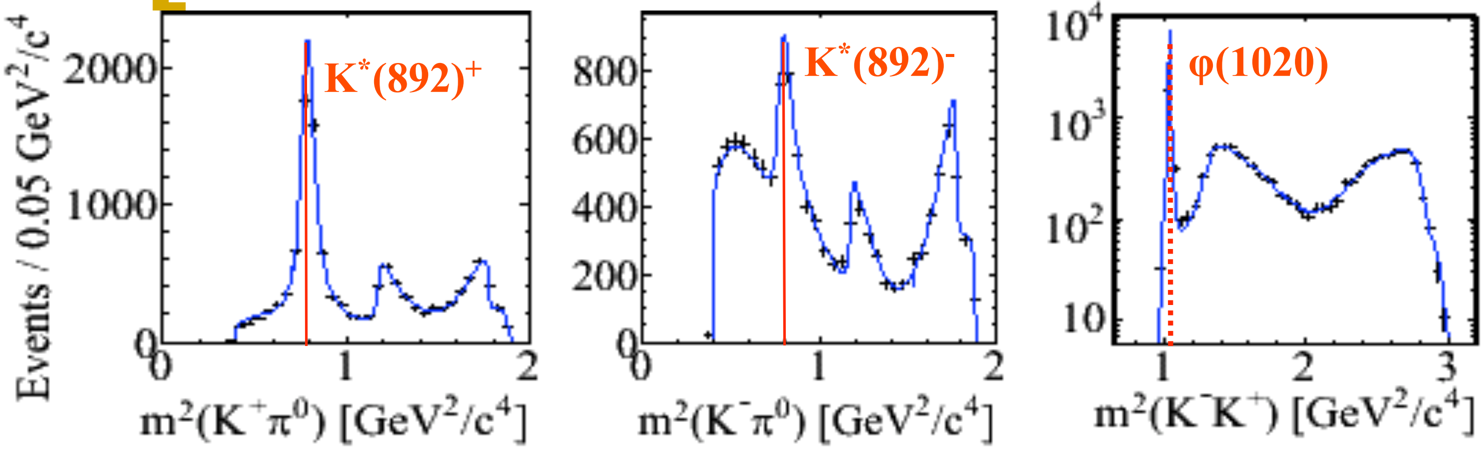
Use LASS amplitude for nominal fit and E-791 amplitude for syst. uncertainty.

For K^-K^+ S-wave

- $f_0(980)$ and $a_0(980)$ virtually indistinguishable from each other.
- Both $f_0(980)$ and $a_0(980)$ give satisfactory fits.

Since they are so similar, we try each as a description of the $K\bar{K}$ S-wave amplitude.

Fit Results for $D^0 \to K^- K^+ \pi^0$ Dalitz Plot



K^{*+} : 45 %
K^{*-} : 16 %
ϕ : 19 %
f_0/a_0: 7-10%

Ambiguity between a large $K^+\pi^0$ S-wave & $K^*(1410)$, $f_2'(1525)$.

	Model-I, $\chi2$ Prob = 62 %			Model-II, $\chi2$ Prob = 48 %		
State	Amplitude, a_r	Phase, ϕ_r (°)	Fraction, f_r (%)	Amplitude, a_r	Phase, ϕ_r (°)	Fraction, f_r (%)
$K^*(892)^+$	1.0 (fixed)	0.0 (fixed)	45.2±0.8±0.6	1.0 (fixed)	0.0 (fixed)	44.4±0.8±0.6
$K^*(1410)^+$	2.29±0.37±0.20	86.7±12.0±9.6	3.7±1.1±1.1			
$K^+\pi^0(S)$	1.76±0.36±0.18	-179.8±21.3±12.3	16.3±3.4±2.1	3.66±0.11±0.09	-148.0±2.0±2.8	71.1±3.7±1.9
$\phi(1020)$	0.69±0.01±0.02	-20.7±13.6±9.3	19.3±0.6±0.4	0.70±0.01±0.02	18.0±3.7±3.6	19.4±0.6±0.5
$f_0(980)$	0.51±0.07±0.04	-177.5±13.7±8.6	6.7±1.4±1.2	0.64±0.04±0.03	-60.8±2.5±3.0	10.5±1.1±1.2
[$a_0(980)^0$]	[0.48±0.08±0.04]	[-154.0±14.1±8.6]	[6.0±1.8±1.2]	[0.68±0.06±0.03]	[-38.5±4.3±3.0]	[11.0±1.5±1.2]
$f_2'(1525)$	1.11±0.38±0.28	-18.7±19.3±13.6	0.08±0.04±0.05			
$K^*(892)^-$	0.601±0.011±0.011	-37.0±1.9±2.2	16.0±0.8±0.6	0.597±0.013±0.009	-34.1±1.9±2.2	15.9±0.7±0.6
$K^*(1410)^-$	2.63±0.51±0.47	-172.0±6.6±6.2	4.8±1.8±1.2			
$K^-\pi^0(S)$	0.70±0.27±0.24	133.2±22.5±25.2	2.7±1.4±0.8	0.85±0.09±0.11	108.4±7.8±8.9	3.9±0.9±1.0

Partial Wave Analysis in K^-K^+ channel

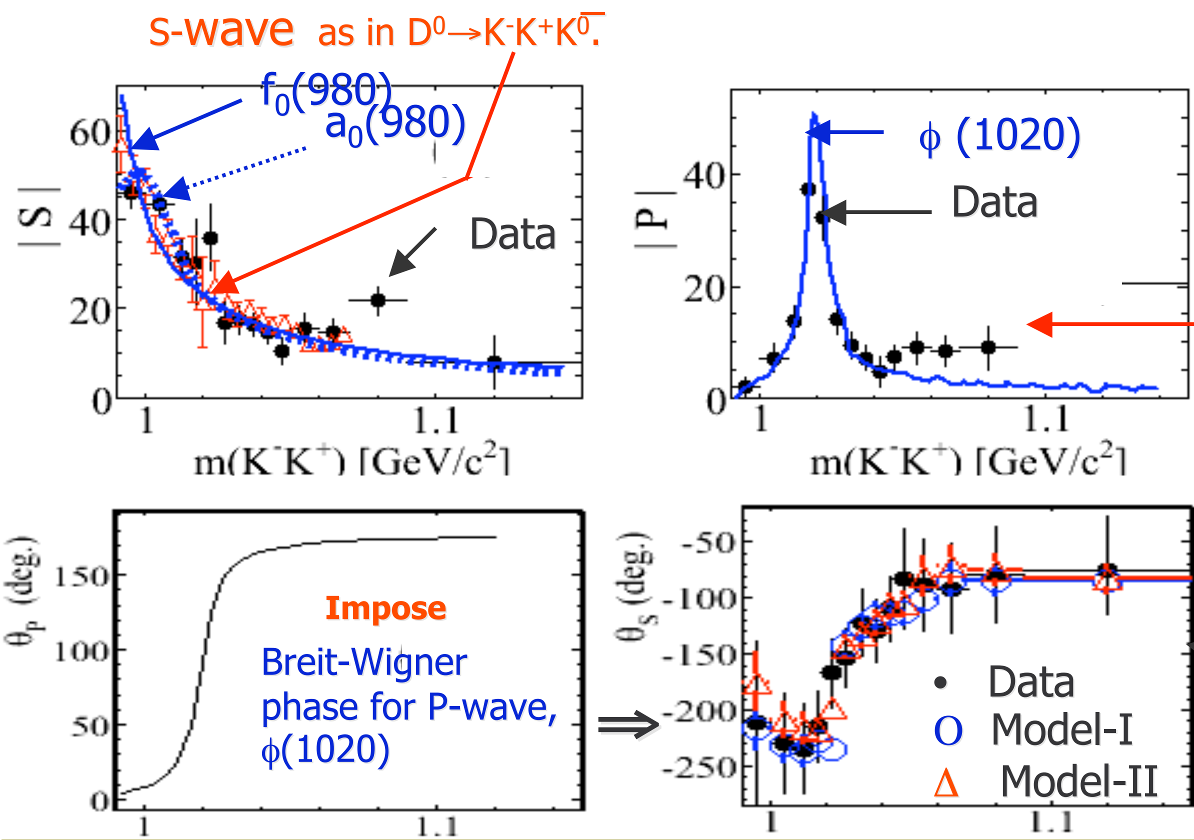
BABAR™

Look into the distributions of the spherical harmonic functions $Y_l^0(\cos\theta_H) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\theta_H)$ (l=0,1,2,.....).

[in the region where the $K\pi$ cross-channels have little effect]

$$\begin{cases} \sqrt{4\pi}\langle Y_0^0 \rangle = S^2 + P^2 \\ \sqrt{4\pi}\langle Y_1^0 \rangle = 2|S||P|\cos\phi_{SP} \\ \sqrt{4\pi}\langle Y_2^0 \rangle = \frac{2}{\sqrt{5}} P^2 \end{cases}$$

Solve these equations to extract |S|, |P|, and cos θ_{SP}.



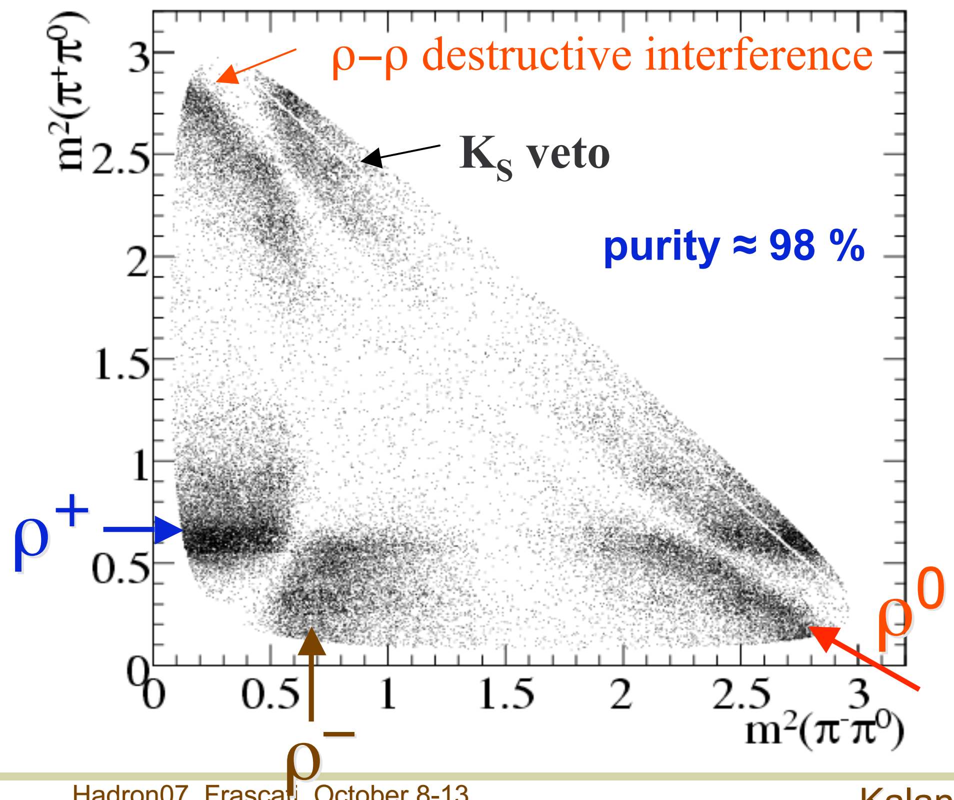
Because of the interference from the crossing $K\pi$ channels, the model independent partial-wave analysis performed here seems valid only up to about 1.02 -1.03 GeV/c^2.

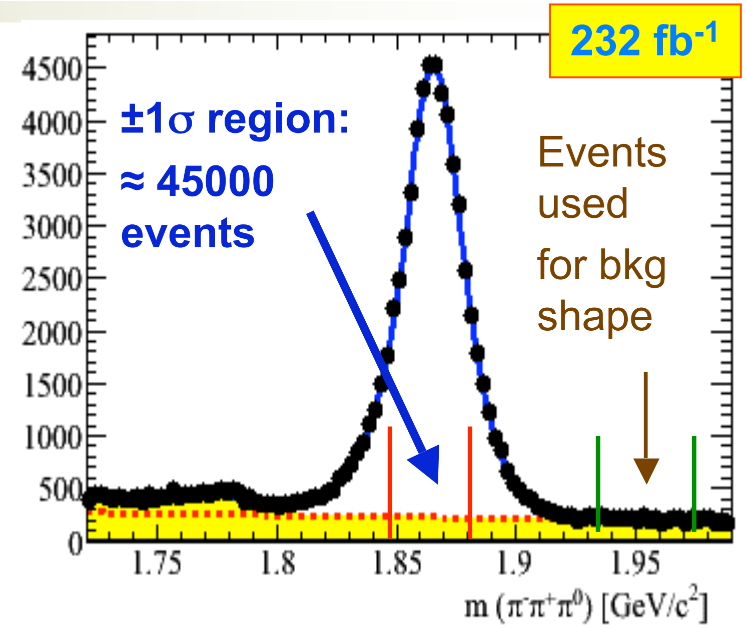
Choose the solution with increasing S-wave phase (Wigner causality)

Dalitz Plot Analysis of $D^0 \rightarrow \pi^- \pi^+ \pi^0$

Motivation: **CKM angle γ using $B^\pm \rightarrow D[\rightarrow \pi^- \pi^+ \pi^0] K^\pm$**

- Three $I = 1$ particles in the final state
- Gives rise to a rich interference structure
- The three ρ regions are clearly enhanced in the DP, and ρ-ρ destructive interference is evident





The 3 destructively interfering $\rho\pi$ amplitudes suggest an $I = 0$, $\Delta I = 1/2$ dominated final state.
C. Zemach, Phys. Rev. 133, B1201 (1964).

hep-ex / 0703037

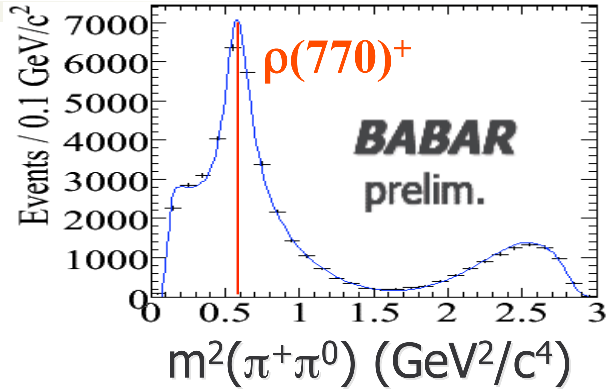
Fit Results for $D^0 \to \pi^- \pi^+ \pi^0$ Dalitz Plot

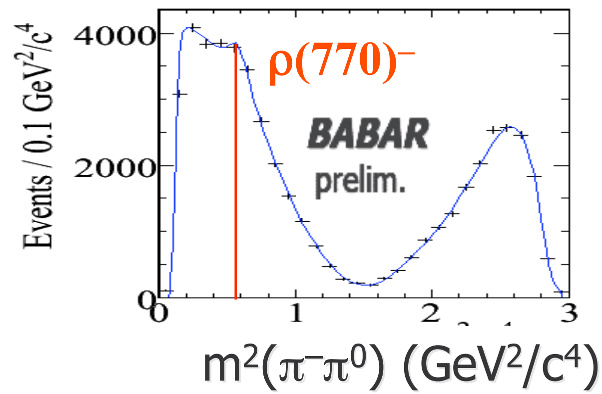
BaBar Preliminary

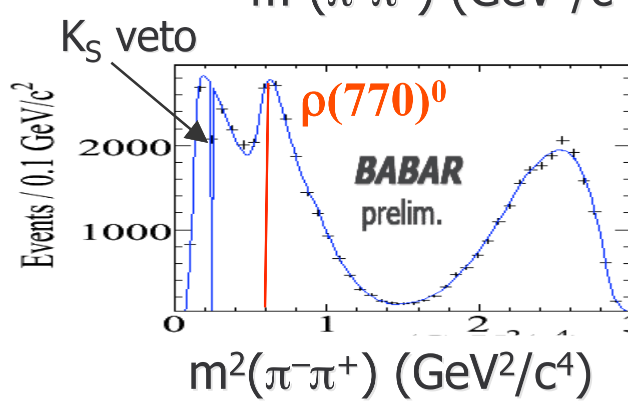
ρ^+ : 68 %
ρ^- : 35 %
ρ^0 : 26 %

Small contributions from higher ρ, f_0, f_2 and σ states

State	Amplitude a_r	Phase ϕ_r	Fraction f_r(%)
$\rho^+(770)$	1	0	67.8±0.0±0.2
$\rho^0(770)$	0.588±0.006±0.001	16.2±0.6±0.3	26.2±0.5±0.4
$\rho^-(770)$	0.714±0.008±0.003	-2.0±0.6±0.5	34.6±0.8±0.1
$\rho^+(1450)$	0.21±0.06±0.10	-146±18±8	0.11±0.07±0.06
$\rho^0(1450)$	0.33±0.06±0.04	10±8±6	0.30±0.11±0.07
$\rho^-(1450)$	0.82±0.05±0.04	16±3±3	1.79±0.22±0.12
$\rho^+(1700)$	2.25±0.18±0.14	-17±2±2	4.1±0.7±0.7
$\rho^0(1700)$	2.51±0.15±0.13	-17±2±2	5.0±0.6±0.9
$\rho^-(1700)$	2.00±0.11±0.07	-50±3±3	3.2±0.4±0.6
$f_0(980)$	0.052±0.004±0.006	-59±5±3	0.25±0.04±0.04
$f_0(1370)$	0.22±0.03±0.03	156±9±6	0.37±0.11±0.09
$f_0(1500)$	0.20±0.02±0.02	12±9±4	0.39±0.08±0.07
$f_0(1710)$	0.39±0.05±0.06	51±8±7	0.31±0.07±0.08
$f_2(1270)$	0.30±0.01±0.06	-171±3±2	1.32±0.08±0.08
σ (400, 600)	0.24±0.02±0.04	8±4±3	0.82±0.10±0.10
Non-Res	0.57±0.07±0.08	-11±4±2	0.84±0.21±0.12



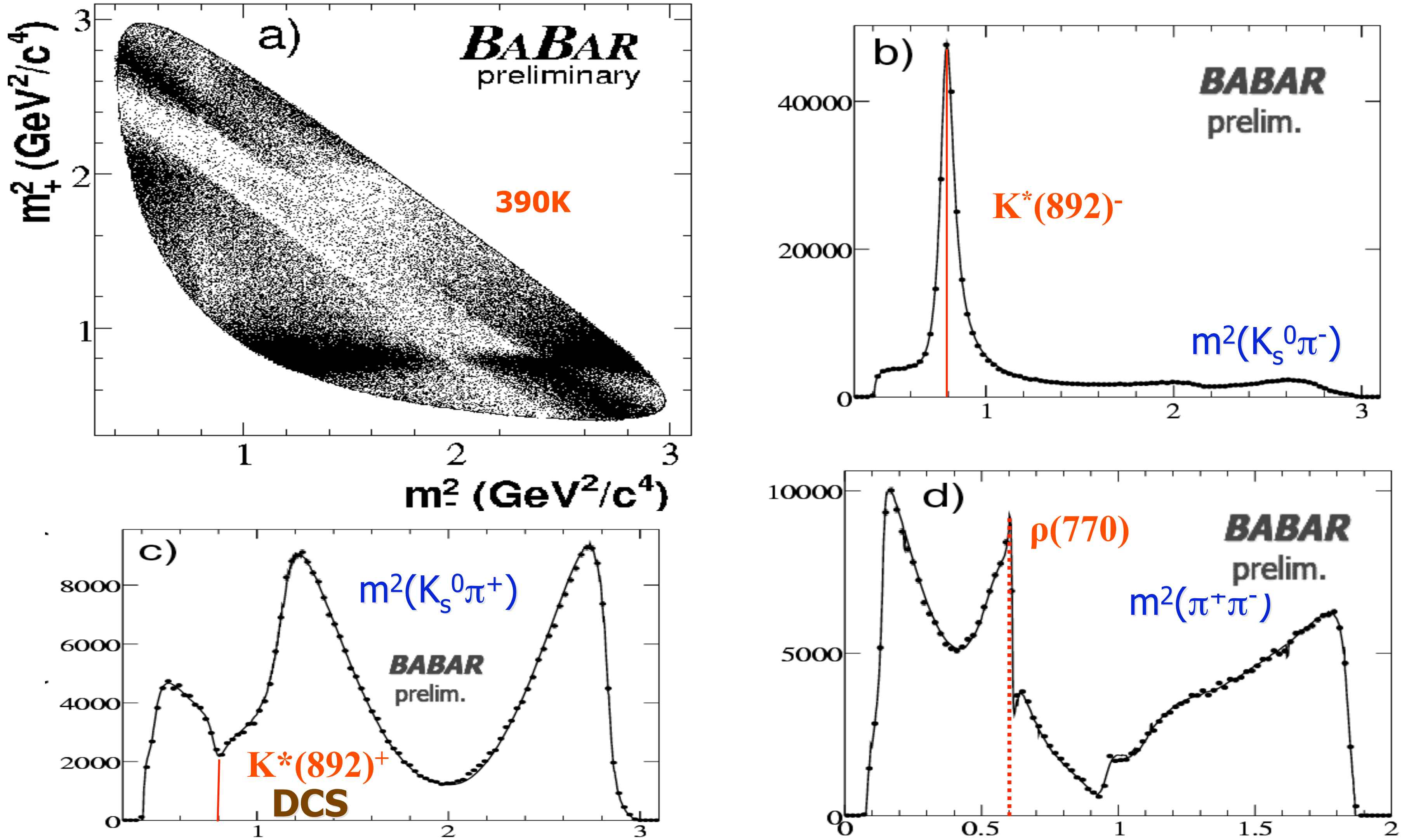




$D^0 \rightarrow K_S^0\pi^+\pi^-$ Dalitz Plot analysis

Motivation: **CKM angle γ using $B \rightarrow D[K_S^0\pi^+\pi^-]K^-$; $D\,\overline{D}$ mixing**

270 fb^{-1}



$D^0 \to K_s^0 \pi^+ \pi^-$ (Isobar Model) Fit

BABAR

Component	$Re\{a_r e^{i\phi_r}\}$	$Im\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$	-1.223 ± 0.011	1.3461 ± 0.0096	58.1
$K_0^*(1430)^-$	-1.698 ± 0.022	-0.576 ± 0.024	6.7
$K_2^*(1430)^-$	-0.834 ± 0.021	0.931 ± 0.022	3.6
$K^*(1410)^-$	-0.248 ± 0.038	-0.108 ± 0.031	0.1
$K^*(1680)^-$	-1.285 ± 0.014	0.205 ± 0.013	0.6
$K^*(892)^+$ DCS	0.0997 ± 0.0036	-0.1271 ± 0.0034	0.5
$K_0^*(1430)^+$ DCS	-0.027 ± 0.016	-0.076 ± 0.017	0.0
$K_2^*(1430)^+$ DCS	0.019 ± 0.017	0.177 ± 0.018	0.1
$\rho(770)$	1	0	21.6
$\omega(782)$	-0.02194 ± 0.00099	0.03942 ± 0.00066	0.7
$f_2(1270)$	-0.699 ± 0.018	0.387 ± 0.018	2.1
$\rho(1450)$	0.253 ± 0.038	0.036 ± 0.055	0.1
Non-resonant	-0.99 ± 0.19	3.82 ± 0.13	8.5
$f_0(980)$	0.4465 ± 0.0057	0.2572 ± 0.0081	6.4
$f_0(1370)$	0.95 ± 0.11	-1.619 ± 0.011	2.0
σ (490, 406)	1.28 ± 0.02	0.273 ± 0.024	7.6
σ'(1024, 89)	0.290 ± 0.010	-0.0655 ± 0.0098	0.9

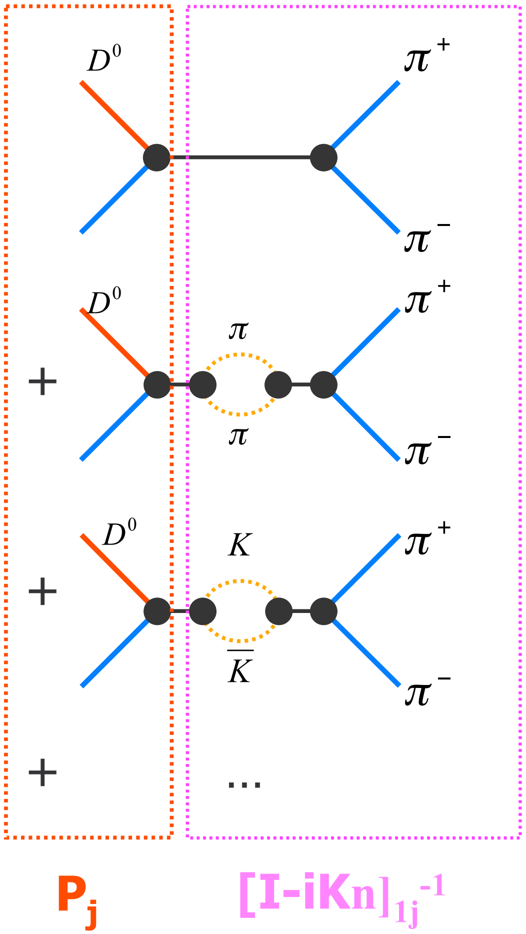
$K^*(892)^-$: 58 %
$\rho(770)^0$: 22 %
Non-Res.: 8 %
σ (500): 8 %
$K^*(1430)^-$: 7 %
$f_0(980)$: 6 %

← Important for γ and D-mixing measurements

Preliminary

hep-ex / 0607104

K-Matrix Model in ππ *S*-wave



K-Matrix formalism overcomes the main limitation of the BW model to parameterize large and overlapping *S*-wave ππ resonances.

$D^0 \to \bar{K}^0 \pi^+ \pi^-$ amplitude

$$\mathcal{A}_D(s_{12}, s_{13}) = \underbrace{F_1}_{\pi\pi\ S-wave} + \underbrace{\sum_r a_r e^{i\delta_r} A_r(s_{12}, s_{13})}_{\substack{\pi\pi\ P,D-waves \\ K\pi\ S,P,D-waves}}$$

$$F_1 = \sum_j [I - iK\rho]^{-1}_{1j} P_j$$

I.J.R. Aitchison, Nucl. Phys. **A189**, 417 (1972)

- P_j → Initial production vector
- $[I - iK\rho]^{-1}_{1j}$ → Provided by scattering experiment

$$P_j(s) = \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - s} + f_{1j}^{\text{prod}} \frac{1 - s_0^{\text{scatt}}}{s - s_0^{\text{scatt}}} .$$

5 channels: **1**=ππ **2**=KK **3**=multi-meson **4**= ηη **5**= ηη´
V.V. Anisovitch, A.V Sarantev Eur. Phys. Jour. **A16**, 229 (2003)

Fit for β_1, β_2, β_4, and f_{11} in this analysis.

$D^0 \rightarrow K_s^0 \pi^+ \pi^-$ (K-Matrix Model) Fit

BABAR™

92 fb^{-1}

Component		$\mathrm{Re}\{a_r e^{i\phi_r}\}$	$\mathrm{Im}\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$		-1.159 ± 0.022	1.361 ± 0.020	58.9
$K_0^*(1430)^-$		2.482 ± 0.075	-0.653 ± 0.073	9.1
$K_2^*(1430)^-$		0.852 ± 0.042	-0.729 ± 0.051	3.1
$K^*(1410)^-$		-0.402 ± 0.076	0.050 ± 0.072	0.2
$K^*(1680)^-$		-1.00 ± 0.29	1.69 ± 0.28	1.4
$K^*(892)^+$	DCS	0.133 ± 0.008	-0.132 ± 0.007	0.7
$K_0^*(1430)^+$	DCS	0.375 ± 0.060	-0.143 ± 0.066	0.2
$K_2^*(1430)^+$	DCS	0.088 ± 0.037	-0.057 ± 0.038	0.0
$\rho(770)$		1 (fixed)	0 (fixed)	22.3
$\omega(782)$		-0.0182 ± 0.0019	0.0367 ± 0.0014	0.6
$f_2(1270)$		0.787 ± 0.039	-0.397 ± 0.049	2.7
$\rho(1450)$		0.405 ± 0.079	-0.458 ± 0.116	0.3
β_1		-3.78 ± 0.13	1.23 ± 0.16	–
β_2		9.55 ± 0.20	3.43 ± 0.40	–
β_4		12.97 ± 0.67	1.27 ± 0.66	–
f_{11}^{prod}		-10.22 ± 0.32	-6.35 ± 0.39	–
sum of $\pi^+\pi^-$ S-wave				16.2

P-vector for $\pi\pi$ S-wave term

These values are consistent with the ones obtained from the Isobar Model fit.

Preliminary

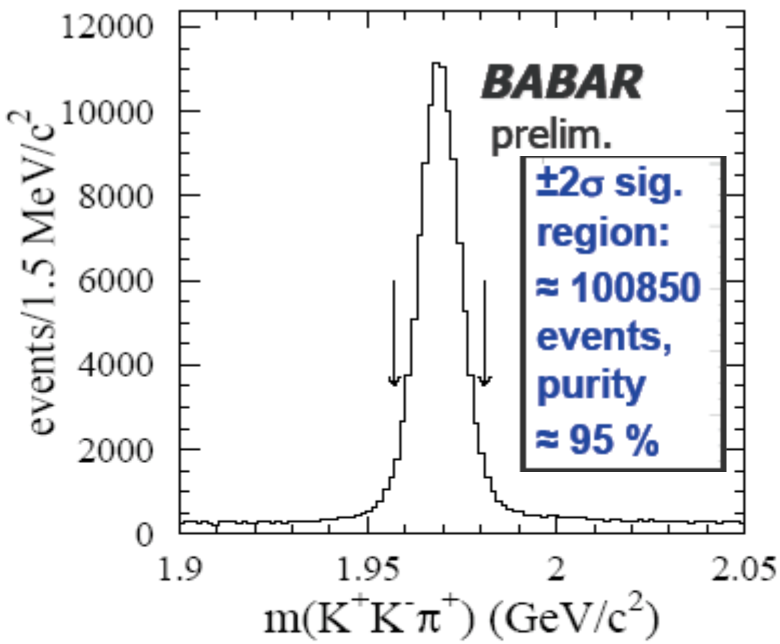
hep-ex/0507101

$D_s^+ \to K^+K^-\pi^+$ Dalitz Plot Analysis

BABAR

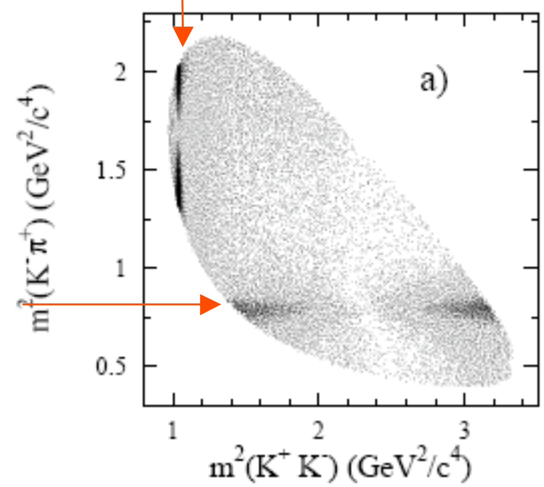
240 fb^{-1}

Preliminary



φ(1020)

$\overline{K}^*(892)^0$



Precise measurement of BR of $D_s^+ \to \varphi\pi^+$ and $D_s^+ \to \overline{K}^*(892)^0K^+$. The $D_s^+ \to \varphi\pi^+$ is frequently used as D_s^+ reference decay mode for measurement of BRs.

Obtained from Dalitz plot analysis

	BaBar Preliminary	PDG 06
$B(D_s^+\to\varphi\pi^+)/B(D_s^+\to K^+K^-\pi^+)$	$= 0.379 \pm 0.002 \pm 0.018$	$0.396 \pm 0.033 \pm 0.047$
$B(D_s^+\to\overline{K}^*(892)^0\pi^+)/B(D_s^+\to K^+K^-\pi^+)$	$= 0.487 \pm 0.002 \pm 0.016$	$0.478 \pm 0.046 \pm 0.040$

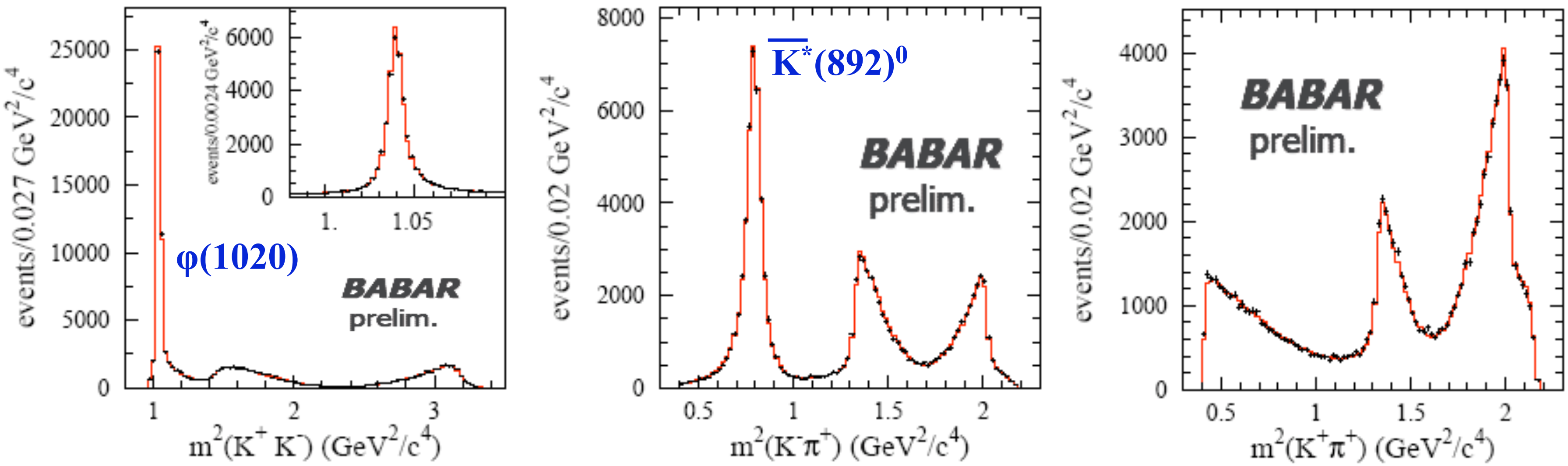
$D_s^+ \to K^+K^-\pi^+$ DP Fit Results

BaBar Preliminary

Decay Mode	Decay fraction(%)	Amplitude	Phase(radians)
$\bar{K}^*(892)^0K^+$	$48.7 \pm 0.2 \pm 1.6$	$1.(Fixed)$	$0.(Fixed)$
$\phi(1020)\pi^+$	$37.9 \pm 0.2 \pm 1.8$	$1.081 \pm 0.006 \pm 0.049$	$2.56 \pm 0.02 \pm 0.38$
$f_0(980)\pi^+$	$35 \pm 1 \pm 14$	$4.6 \pm 0.1 \pm 1.6$	$-1.04 \pm 0.04 \pm 0.48$
$\bar{K}_0^*(1430)^0K^+$	$2.0 \pm 0.2 \pm 3.3$	$1.07 \pm 0.06 \pm 0.73$	$-1.37 \pm 0.05 \pm 0.81$
$f_0(1710)\pi^+$	$2.0 \pm 0.1 \pm 1.0$	$0.83 \pm 0.02 \pm 0.18$	$-2.11 \pm 0.05 \pm 0.42$
$f_0(1370)\pi^+$	$6.3 \pm 0.6 \pm 4.8$	$1.74 \pm 0.09 \pm 1.05$	$-2.6 \pm 0.1 \pm 1.1$
$\bar{K}_2^*(1430)^0K^+$	$0.17 \pm 0.05 \pm 0.3$	$0.43 \pm 0.05 \pm 0.34$	$-2.5 \pm 0.1 \pm 0.3$
$f_2(1270)\pi^+$	$0.18 \pm 0.03 \pm 0.4$	$0.40 \pm 0.04 \pm 0.35$	$0.3 \pm 0.2 \pm 0.5$
Sum	$132 \pm 1.2 \pm 15.6$		
χ^2/NDF	1.5		

$K^*(892)^0$: 49 %
$\phi(1020)$: 38 %
$f_0(980)$: 35 %

- Decay dominated by P wave
- $f_0(980)$ contribution is large but big syst. uncertaínces



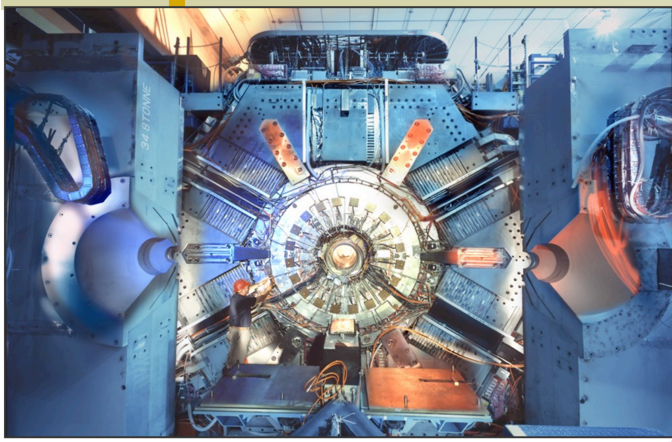
Summary

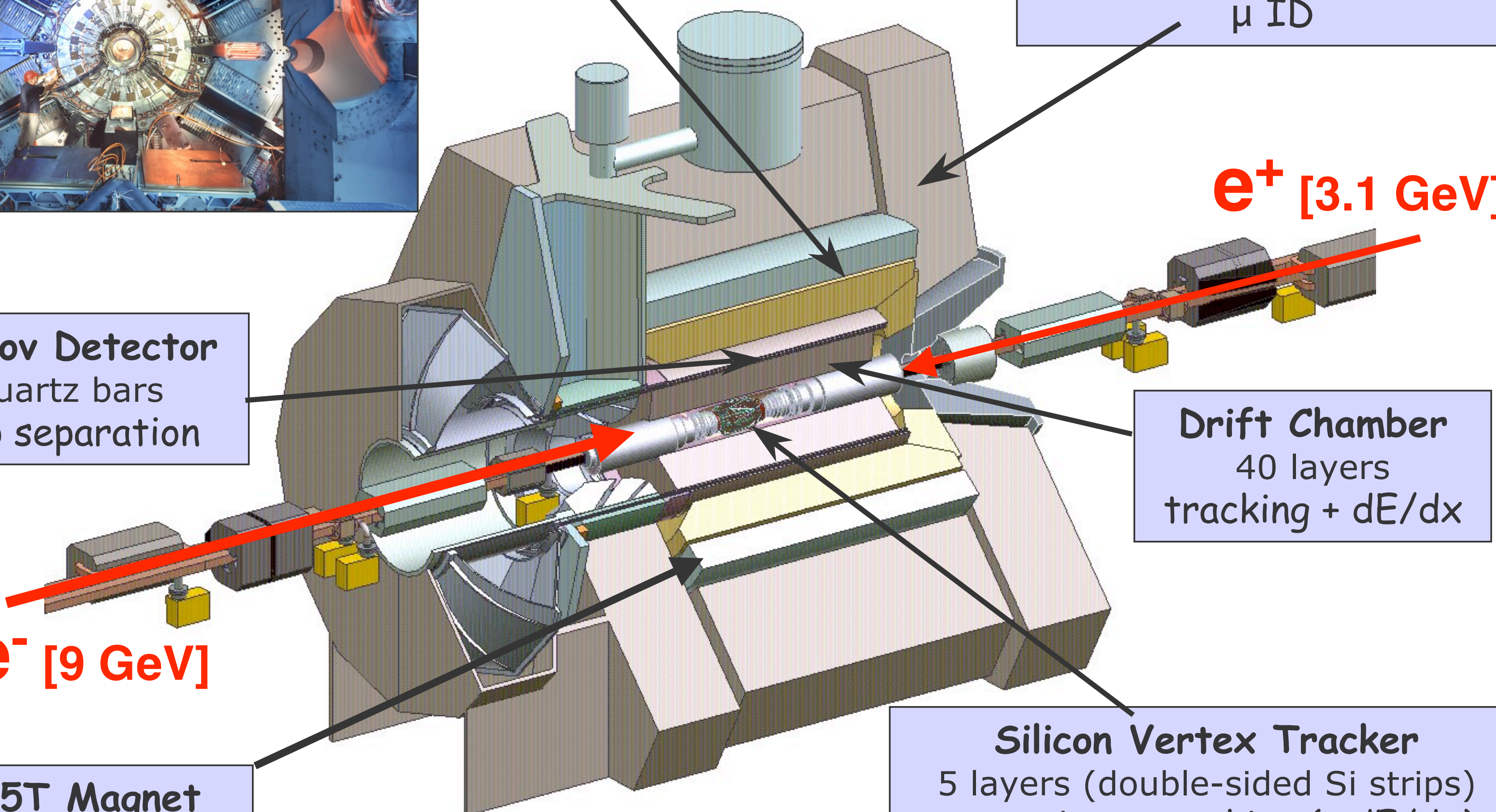
- Precision measurement of the amplitude structure of $D^0 \to K^- K^+ \pi^0$ Dalitz plot; also measure K^+K^- *S*-wave amplitude in a model-independent way upto ~ 1.02–1.03 GeV/c^2.
  - No evidence for charged κ states.
- Dalitz plot analysis of $D^0 \to \pi^- \pi^+ \pi^0$ gives information on the $\rho\pi$ amplitude contributions to the final state.
- Rich resonant structure of $D^0 \to K_S^0 \pi^- \pi^+$ Dalitz plot investigated in two different ways; a consistent and satisfactory parameterization of the amplitudes accomplished.
- Dalitz plot analysis of the decay $D_s^+ \to K^+ K^- \pi^+$ performed; precise branching ratio information on the $\phi\pi^+$ and $\overline{K^{*0}}K^+$ decays obtained.
- Some of these analyses will benefit from higher statistics; we will update them as our data size grows.

End of Talk ! Thank You !

Back up slides

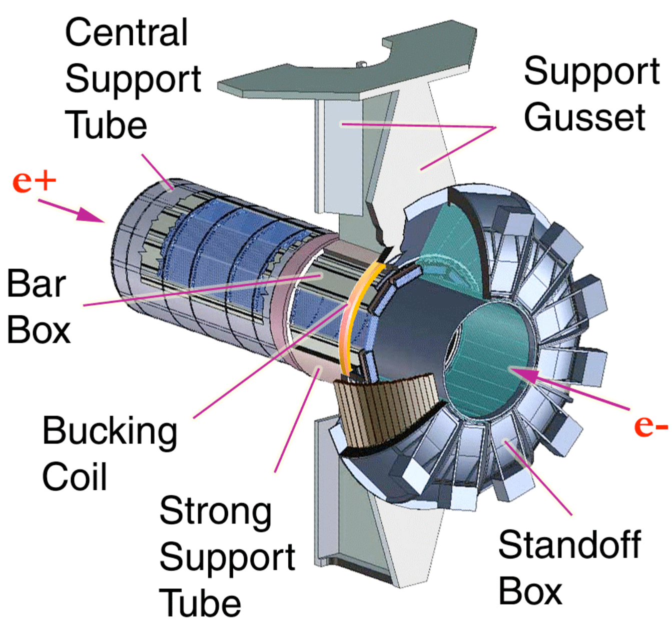
BaBar: B and charm Factory



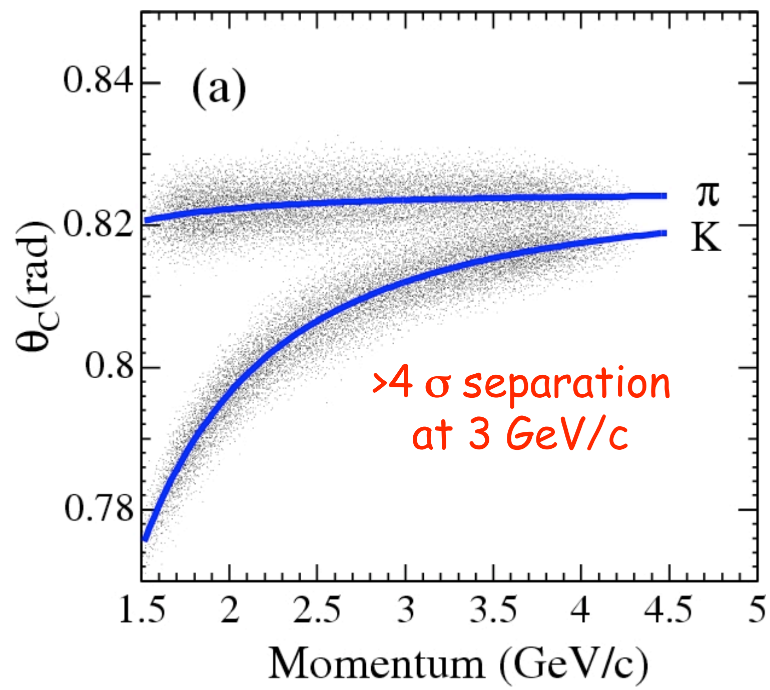


Kaon/Pion Discrimination: DIRC

LAYOUT



Cherenkov angle vs. momentum for pions and kaons



$D^0 \to \pi^- \pi^+ \pi^0$, $K^- K^+ \pi^0$

$D^0 \to h^- h^+ \pi^0$ Reconstruction

- h^- and h^+ tracks are fit to a vertex
- Mass of π^0 candidate is constrained to m_{π^0} at $h^- h^+$ vertex
- P_{CM} (D^0) > 2.77 GeV/c

Background Sources

- Charged track combinatoric
- Mis-reconstructed π^0
- Real D^0, fake π_s
- $K\pi\pi^0$ reflection in $\pi\pi\pi^0$ and $KK\pi^0$ modes

D* Reconstruction

- D^{*+} candidate is made by fitting the D^0 and the π_s^+ to a vertex constrained in x and y to the measured beam-spot for the run.
- $|m_{D^*} - m_{D^0} - 145.5| < 0.6$ MeV/c^2
- Vertex χ^2 probability > 0.01
- Choose a single best candidate with smallest χ^2 for the whole decay chain (multiplicity = 1.03).

$KK\pi^0$: Strong-phase Diff. & Amp. Ratio

- The strong phase difference δ_D and relative amplitude r_D between the decays of D^0 and D^0 to $K^*(892)^+ K^-$ state are defined, neglecting direct CP violation in D decays, by the equation:

$$r_D e^{i\delta_D} = \frac{a_{D^0 \to K^{*-}K^+}}{a_{D^0 \to K^{*+}K^-}} e^{i(\delta_{K^{*-}K^+} - \delta_{K^{*+}K^-})}$$

- We find

BaBar	CLEO
$r_D = 0.599 \pm 0.013$ (stat) ± 0.011 (syst) $\delta_D = -35.5°$ (stat) $\pm 1.9° \pm 2.2°$ (syst)	$r_D = 0.52 \pm 0.05$ (stat) ± 0.04 (syst) $\delta_D = -28° \pm 8°$ (stat) $\pm 11°$(syst)
Phys. Rev. D76, 011102 (2007)	Phys. Rev. D74, 031108 (2006)

These measurements are consistent with each other.

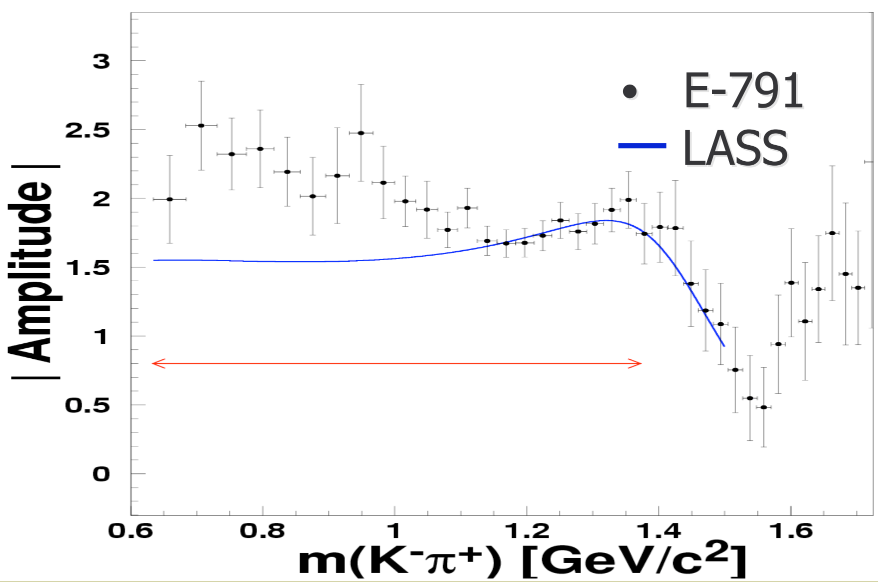
$KK\pi^0$: $K\pi$ S-wave Parametrization

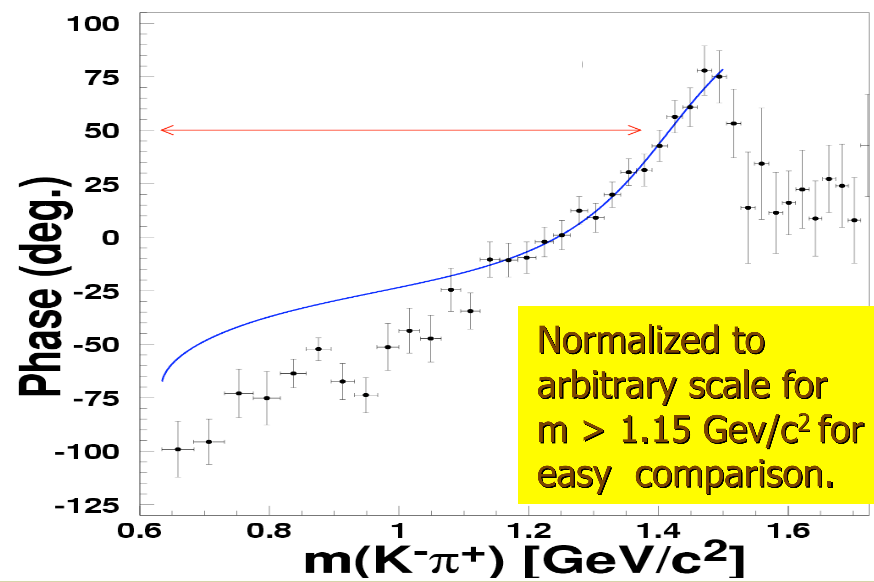
- $K\pi$ S-wave in mass range 0.6–1.4 GeV/c^2 is not well-understood.
- A possible κ state ~ 800 MeV/c^2 has been conjectured, but has only been reported in the neutral state.
- For the $K^+\pi^0$ and $K^-\pi^0$ S-wave amplitudes, we try three models:

- Amplitude from LASS $K^-\pi^+ \to K^-\pi^+$ scattering. Nucl. Phys. B296, 493 (1988);

- $K^-\pi^+$ amplitude from a model-independent analysis of $D^+ \to K^-\pi^+\pi^+$ data by the E791 collaboration. Phys. Rev. D73, 032004 (2006);

- [coherent sum of κ(800) + uniform NR + K^*_0(1430)]. {No evidence in $K\pi$ elastic scattering.}





LASS Kπ S-wave Parameterization

Kπ S-wave amplitude is described by the coherent sum of an effective range term and the $K^*_0(1430)$ resonance:

$$S(s) = (\sqrt{s}/p) \,.\, \sin\Delta \,.\, e^{i\Delta}$$

Phase space factor

$$\Delta = \cot^{-1}[\,1/ap + rp/2\,] + \cot^{-1}[\,(m^2_R - s)/(m_R \Gamma_R)\,]$$

Effective Range (NR) term — **$K^*_0(1430)$ resonance term**

a = scat. length, r = eff. range, m_R = mass of $K^*_0(1430)$, Γ_R= width
p = momentum of either daughter in the Kπ rest frame.

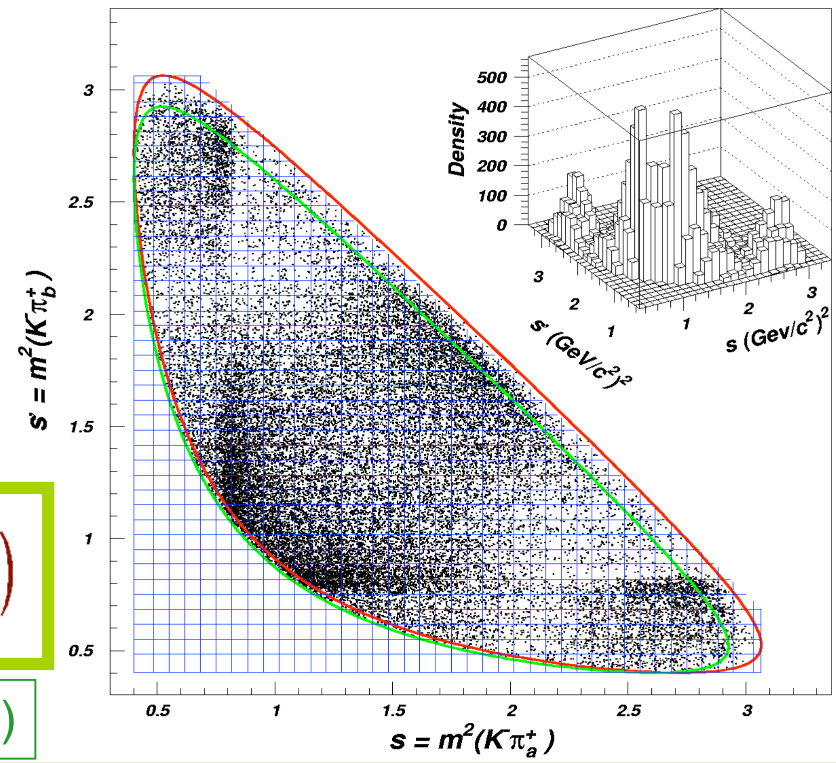
For Kπ scattering, S-wave is elastic up to Kη' threshold (1.45 GeV).

$K\pi$ S-wave from $D^0 \to K^-\pi^+\pi^+$ DP

[E791 Collaboration, slide from Brian Meadow's Moriond 2005 talk]

- Divide $m^2(K^-\pi^+)$ into slices
- Find s–wave amplitude in each slice (two parameters)
  - Use remainder of Dalitz plot as an interferometer

$$\frac{d^2\Gamma}{ds_{12}ds_{13}} \propto \left|\mathcal{S} + (\mathcal{P} + \mathcal{D})\right|^2$$



- For s-wave:

\- Interpolate between (c_k, γ_k).

- Model P and D waves.

$$\mathcal{S} = \mathrm{Interp}\left(c_k e^{i\gamma_k}\right) \times F_0^D(q, r_D) F_0^R(p, r_R)$$

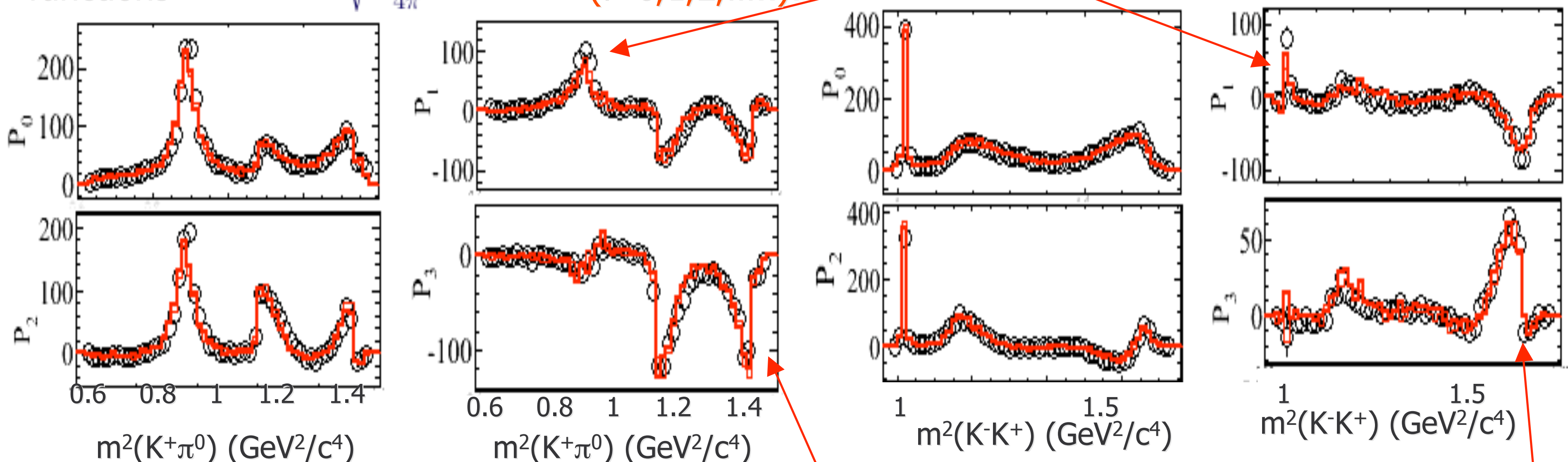
S ("partial wave")

Analysis of Angular Moments

Each event is weighted by the spherical harmonic functions $Y_l^0(\cos\theta_H) = \sqrt{\frac{2l+1}{4\pi}}\,P_l(\cos\theta_H)$ (l=0,1,2,.....).

Excellent agreement between data & models.

Large interference between S and P waves.



Higher moments above 1.1 GeV are coming from cross channels.

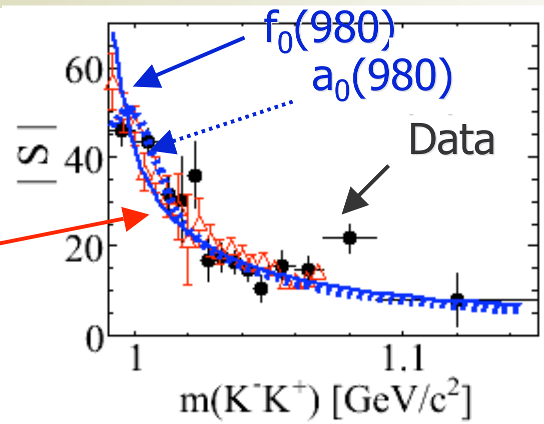
$$\begin{cases} \sqrt{4\pi}\left\langle Y_0^0 \right\rangle = S^2 + P^2 \\ \sqrt{4\pi}\left\langle Y_1^0 \right\rangle = 2|S||P|\cos\phi_{SP} \\ \sqrt{4\pi}\left\langle Y_2^0 \right\rangle = \frac{2}{\sqrt{5}} P^2 \end{cases}$$

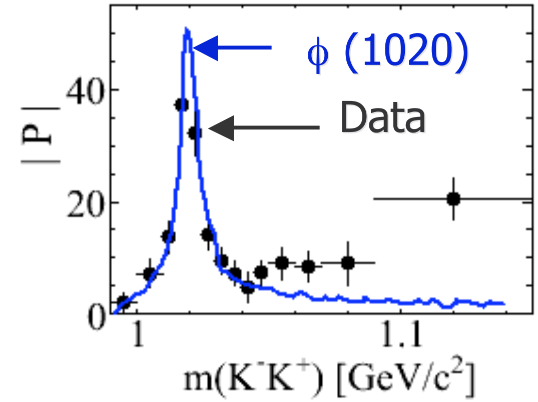
For S- and P- waves only, in the absence of cross-feeds from other channels, the amplitudes and the relative phase are given by:

We solve these equations for the K^-K^+ system in a limited mass range (where the above conditions are satisfied) to extract |S|, |P|, and cos ϕ_{SP}.

Partial Wave Analysis in K^-K^+ channel

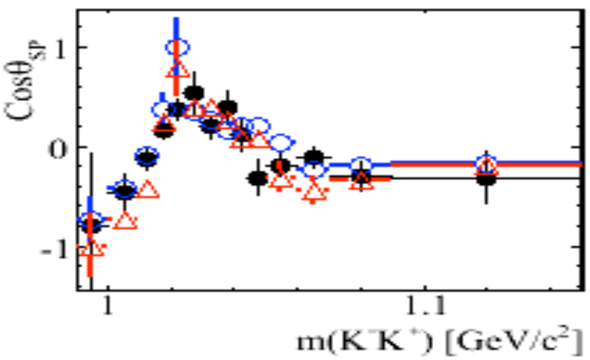
Solve the equations on the previous slide to extract |S|, |P|, and cos θ_{SP}.

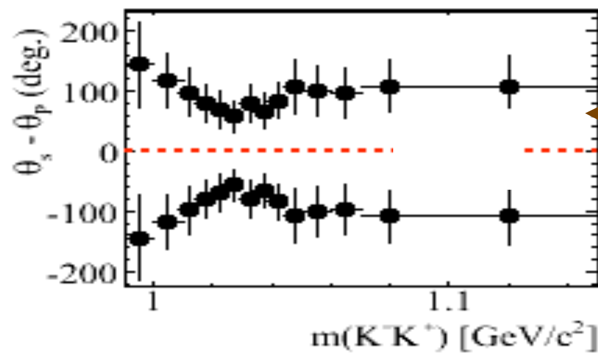




KK S-wave amplitude, extracted in a model independent analysis of the decay $D^0 \to K^-K^+\overline{K}^0$.

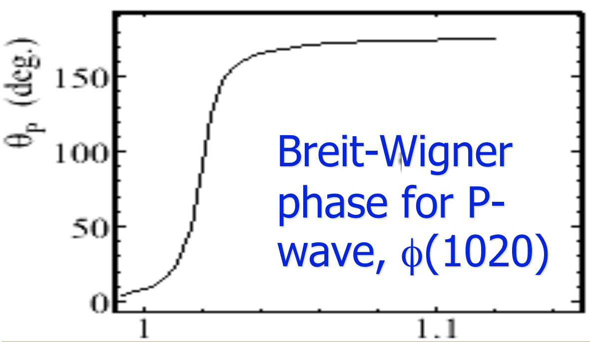
Phys. Rev. D 72, 052008 (2005)

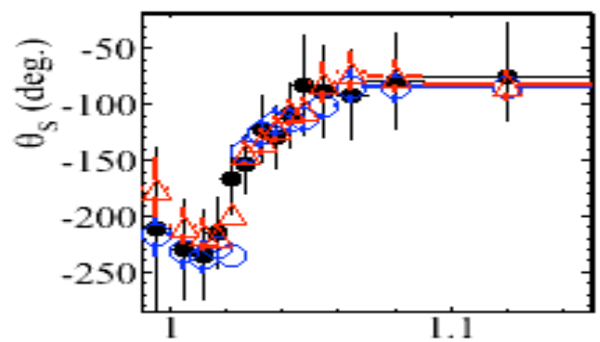




Two solutions for $\theta_{SP} \Rightarrow$ upper one is the physical

Because of the interference from the crossing $K\pi$ channels, the model independent partial-wave analysis performed here is valid only up to about 1.02 -1.03 GeV/c^2





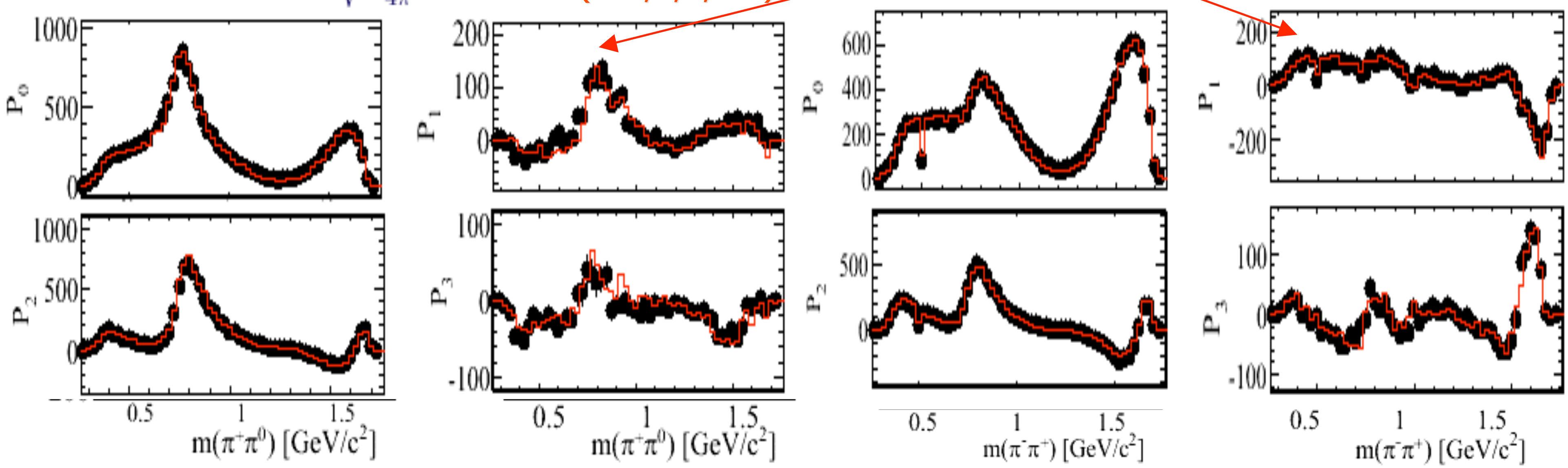
- • Data
- O Model-I
- Δ Model-II

$D^0 \rightarrow \pi^- \pi^+ \pi^0$: Angular Moments

Each event is weighted by the spherical harmonic functions $Y_l^0(\cos\theta_H) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\theta_H)$ (l=0,1,2,…..).

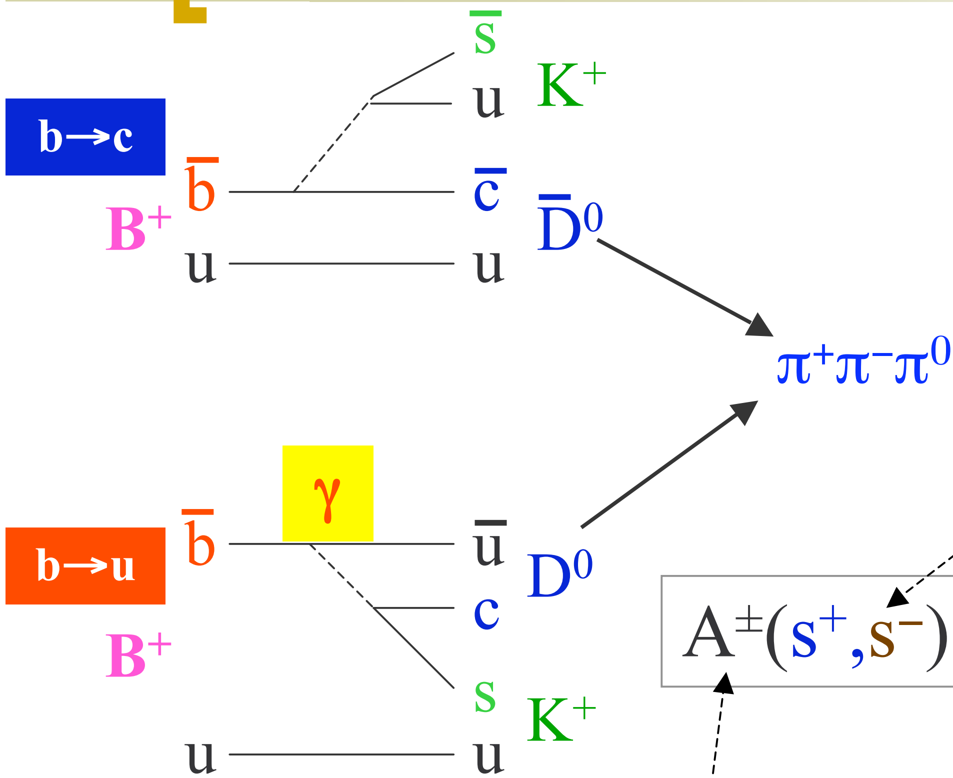
Excellent agreement between data & fit.

Large interference between S and P waves.



Extraction of γ : Basic Idea

Atwood et al., PRL78, 3257 (1997)
Giri et al., PRD68,054018 (2003)



- δ_B is strong phase diff. of $A(B^- \to D^0K^-)$ and $A(B^- \to \bar{D}^0K^-)$
- Unknowns: r_B, δ_B and γ

Dalitz plot variables

$$A^{\pm}(s^+,s^-) = f_D(s^+,s^-) + r_B e^{i(\delta\pm\gamma)} f_D(s^-,s^+)$$

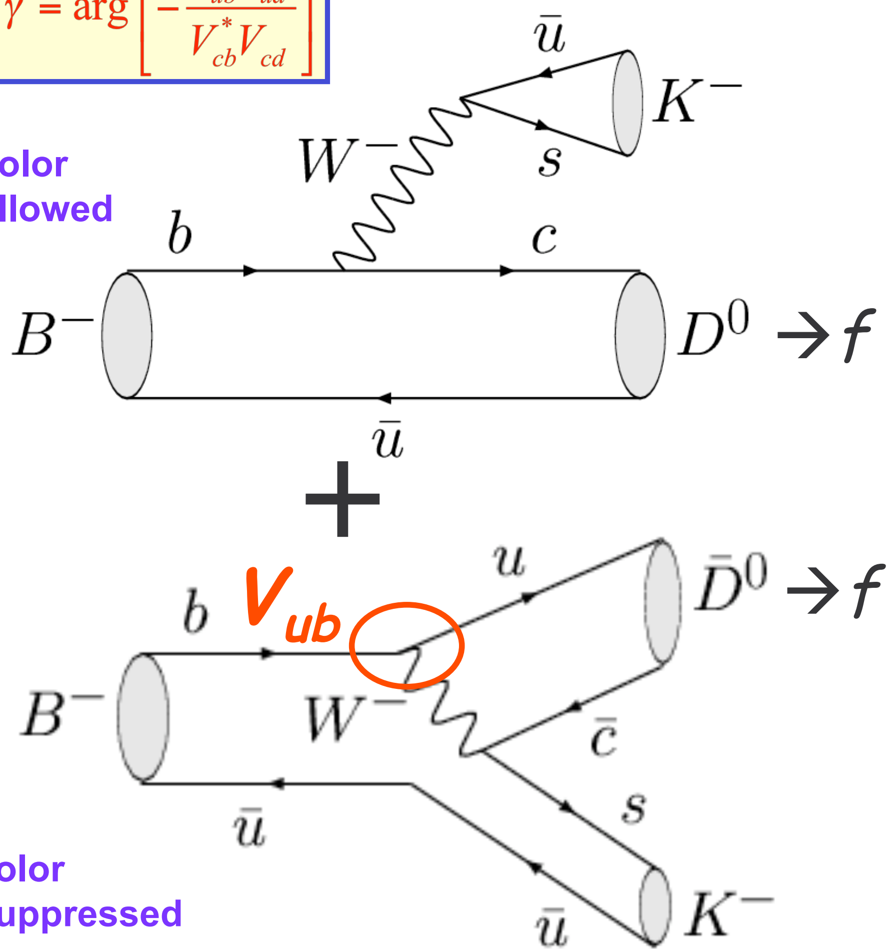
$B^{\pm}\to D_{\pi\pi\pi^0}K^{\pm}$ amplitude — $\bar{D}^0\to\pi\pi\pi^0$ amplitude — $D^0\to\pi\pi\pi^0$ amplitude

- Based on GGSZ method of **PRD68, 054018**, so far used only with $D\to K_S\pi^+\pi^-$
- Goal: add modes for maximum γ precision

Extraction of γ with B→D^0K

$$\gamma = \arg\left[-\frac{V_{ub}^{*}V_{ud}}{V_{cb}^{*}V_{cd}}\right]$$

color allowed



color suppressed

- $D^0/\overline{D}^0$ decay to common final state
- The interference depends on V_{ub} and therefore on γ
- Critical parameter: ratio of amplitudes:

$$r_B \equiv \left|\frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| \sim 0.1$$

- Select the D^0 decays that enhance the interference:
  - o 3-body (e.g. $K_S\pi\pi$): **Dalitz**
  - o CP-eigen. (e.g. $K_S\pi^0$): **GLW**
  - o DCS (e.g. $D^0 \to K^+\pi^-$): **ADS**

K-Matrix for ππ *S*-wave

$$\mathcal{A}_D(m_-^2, m_+^2) = F_1(s) + \Sigma_{r\neq\pi\pi\,\mathrm{S-wave}} a_r e^{i\phi_r} \mathcal{A}_r(m_-^2, m_+^2)\ ,$$

$$F_1(s) = \sum_j [I - iK(s)\rho(s)]^{-1}_{1j}\, P_j(s)\ .$$

Here, s is the squared mass of the $\pi\pi$ system ($m^2_{\pi^+\pi^-}$), I is the identity matrix, K is the matrix describing the S-wave scattering process, ρ is the phase-space matrix, and P is the initial production vector

$$P_j(s) = \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - s} + f_{1j}^{\mathrm{prod}} \frac{1 - s_0^{\mathrm{scatt}}}{s - s_0^{\mathrm{scatt}}}\ .$$

The index j represents the j^{th} channel ($1 = \pi\pi$, $2 = K\overline{K}$, $3 =$multi-meson[5], $4 = \eta\eta$, $5 = \eta\eta'$)

$$K_{ij}(s) = \left\{\sum_\alpha \frac{g_i^\alpha g_j^\alpha}{m_\alpha^2 - s} + f_{ij}^{\mathrm{scatt}} \frac{1.0 - s_0^{\mathrm{scatt}}}{s - s_0^{\mathrm{scatt}}}\right\} \frac{(1 - s_{A0})}{(s - s_{A0})}(s - s_A m_\pi^2/2)$$

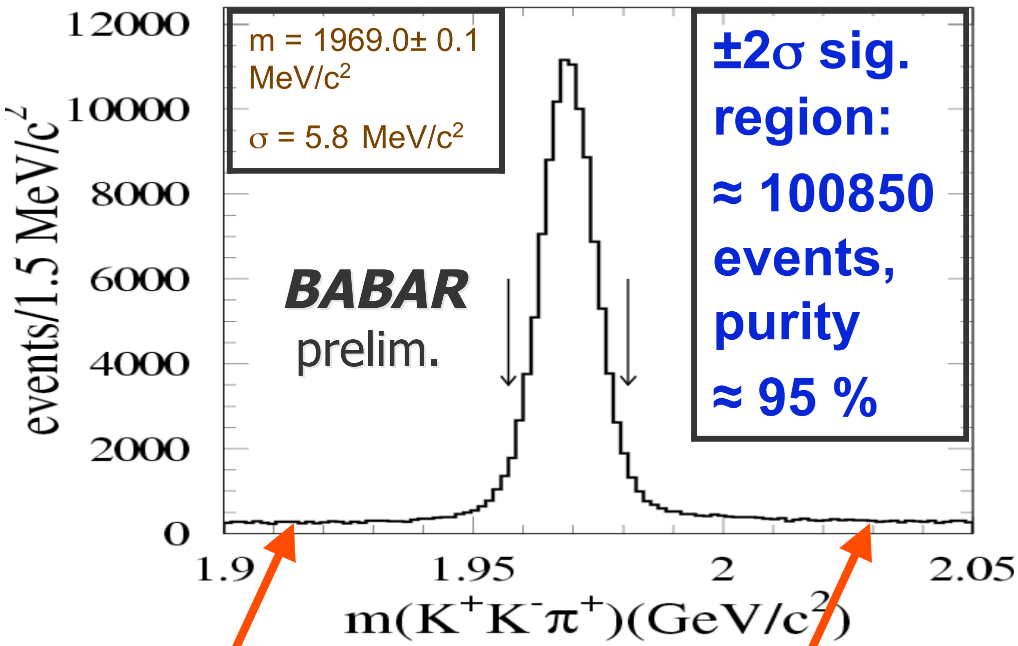
m_α	$g^\alpha_{\pi^+\pi^-}$	$g^\alpha_{K\overline{K}}$	$g^\alpha_{4\pi}$	$g^\alpha_{\eta\eta}$	$g^\alpha_{\eta\eta'}$
0.651	0.229	−0.554	0.000	−0.399	−0.346
1.204	0.941	0.551	0.000	0.391	0.315
1.558	0.369	0.239	0.556	0.183	0.187
1.210	0.337	0.409	0.857	0.199	−0.010
1.822	0.182	−0.176	−0.797	−0.004	0.224

f^{scatt}_{11}	f^{scatt}_{12}	f^{scatt}_{13}	f^{scatt}_{14}	f^{scatt}_{15}
0.234	0.150	−0.206	0.328	0.354

s^{scatt}_0	s_{A0}	s_A
−3.926	−0.15	1

$D_s^+ \to K^+K^-\pi^+$

Data Sample = 240 fb^{-1}

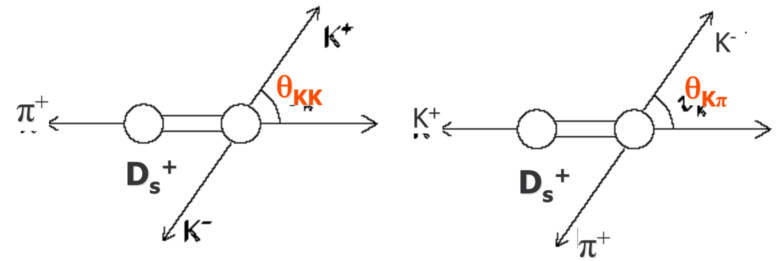


Events used to obtain Bkg shape: (-10σ, -6σ) and (6σ, 10σ).

- Signal events reconstructed from two kaon and a pion charged tracks fitted to a common vertex, with $\chi^2 > 0.1$ %.
- Background from $D^{*+} \to D^0[K^+K^-]\,\pi^+$ removed by requiring $m(K^+K^-) < 1.85$ GeV/c^2.
- Removed $K^-\pi^+_{mis}\pi^+$ reflection by requiring $m(K^-\pi^+_{mis}\pi^+) - m(K^-\pi^+_{mis}) > 0.15$ GeV/c^2.
- Average event reconstruction efficiency ~ 30 %.

$D_s^+ \to K^+K^-\pi^+$ Angular Moments

Each event was weighted by the spherical harmonic $Y^0_\ell(\cos\theta_{KK})$ and $Y^0_\ell(\cos\theta_{K\pi})$ (ℓ =1,2,3,4)



$f_0(980)/\varphi(1020)$ interference

Small S-P interference ⇒ No k(800) ?